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Readers' Forum

Brief discussion of previous investigations in the aerospace sciences and technical comments on papers published in the AIAA Journal *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

Comment on "Application of Wall Functions to Generalized Nonorthogonal Curvilinear Coordinate Systems"

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In the paper by Sondak and Pletcher,[1] the authors argued that the wall function approach of Huang and Coakley[2] is in error because slopes of velocity and enthalpy near the wall "will not be computed correctly." The method of Huang and Coakley is essentially the same as that of Sondak and Pletcher but requires no complex tensorial transformation. The authors have perhaps misinterpreted the methodology discussed in Huang and Coakley, and therefore differences of the two approaches will be compared in this Comment.

Referring to Fig. 1, with u_2, ρ_w, and μ_w evaluated from the previous iteration, the friction velocity, $u_* \equiv \sqrt{(\tau_w/\rho_w)}$, and the wall shear stress, $\tau_w \equiv \rho_w u_*^2$, can be obtained directly from the law-of-the-wall formula. Herein, it is assumed that point 2 is inside the log region ($y^+ > y_c^+$) and the law-of-the-wall formula is

$$u_2^+ = \frac{u_2}{u_*} = \frac{1}{\kappa}\ell n\left[\frac{u_*(y_2 - y_1)\rho_w}{\mu_w}\right] + B \tag{1}$$

In Sondak and Pletcher, Eq. (1) was solved using Newton iteration, and τ_w was incorporated directly into the Navier–Stokes equations. For example, in Cartesian coordinates, as shown in Fig. 1, the diffusion term of the x-momentum equation, $\partial\tau^{xy}/\partial y$, is approximated at point 2 according to

$$\int_{1\frac{1}{2}}^{2\frac{1}{2}} \frac{\partial \tau^{xy}}{\partial y}\,dy \approx \tau^{xy}_{2\frac{1}{2}} - \tau^{xy}_{1\frac{1}{2}} \tag{2}$$

The shear stress at point $1\frac{1}{2}$ is obtained by assuming a constant stress layer near the wall and thus

$$\tau^{xy}_{1\frac{1}{2}} = \tau_w \tag{3}$$

In Huang and Coakley, Eq. (1) was solved by employing the concept of effective viscosity between points 1 and 2. The method is exactly the same as that proposed by Sondak and Pletcher, but the implementation is much simpler. Equation (1) can be reformulated as

$$\tau_w = \rho_w u_* \frac{u_2}{u_2^+} = \mu_{\text{eff}}\frac{u_2 - u_1}{y_2 - y_1} \tag{4}$$

where

$$\mu_{\text{eff}} \equiv \mu_w\left(y_2^+ - y_1^+\right)/u_2^+ \tag{5}$$

being an effective viscosity defined to render the resultant τ_w from Eq. (1) as if a linear velocity profile existed between points 1 and 2 (the choice of the linear velocity profile is only for computational convenience). If a centered difference scheme is used for the diffusion term between points 1 and 2, $\tau^{xy}_{1\frac{1}{2}}$ from Eq. (3) can be put into the Navier–Stokes equation implicitly simply by updating the eddy viscosity at point $1\frac{1}{2}$ according to Eq. (5) (assuming a constant stress layer). It should be noted that the exact value of u_* (or τ_w) resulting from the solution of Eq. (1) is not necessary during iterations because u_2 is inaccurate.

Similarly, the same methodology can be applied in the solution of the energy equation. Assuming a constant Prandtl number Pr_t near the wall, the heat transfer at the wall is written as

$$q_w = -\frac{\mu_{\text{eff}} c_p}{Pr_t}\frac{T_2 - T_1}{y_2 - y_1} - \mu_{\text{eff}}\frac{\left(u_2^2 - u_1^2\right)/2}{y_2 - y_1} \tag{6}$$

where the first term on the right-hand side (RHS) is the heat flux at point $1\frac{1}{2}$, which is treated implicitly. Analogous to the solution of the momentum equations, the heat flux at point $1\frac{1}{2}$ can be put into the energy equation by assuming the effective conductivity at point $1\frac{1}{2}$ to be $\alpha_{\text{eff}} = \mu_{\text{eff}} c_p / Pr_t$. The second term on the RHS of Eq. (6) represents the diffusive transport of the mean kinetic energy between points 1 and 2 and must be treated explicitly. The assumption of a constant turbulent Prandtl number implies that the law of the wall for the temperature and the law of the wall for the velocity are similar. Therefore the same effective viscosity μ_{eff} can be used in both the heat transfer and the momentum equations. A more sophisticated law of the wall for the temperature can be applied, but this should not alter the methodology discussed in this Comment except that the effective heat conductivity has to be evaluated directly from the law of the wall for the temperature.

The advantage of the Huang and Coakley's approach is that it defines a single scalar variable μ_{eff} between points 1 and 2 and makes the implementation of wall functions in generalized curvilinear coordinates easier. Sondak and Pletcher's approach becomes more cumbersome in generalized curvilinear coordinates because it is the shear stress tensor that is put directly into the Navier–Stokes equations. For example, for an inclined wall as shown in Fig. 2, the law of the wall, Eq. (1), defined in the ξ–η coordinate system becomes

$$\hat{u}_2^+ = \frac{\hat{u}_2}{\hat{u}_*} = \frac{1}{\kappa}\ell n\left[\frac{\hat{u}_*(\eta_2 - \eta_1)\rho_w}{\mu_w}\right] + B \tag{7}$$

and $\tau_w \equiv \tau^{\xi\eta} = \rho_w \hat{u}_*^2$, where $\hat{u}$ represents the velocity component parallel to the wall. Since most computational fluid dynamics codes

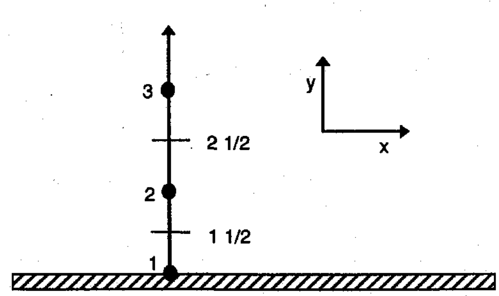

Fig. 1 Simple two-dimensional Cartesian coordinate system.

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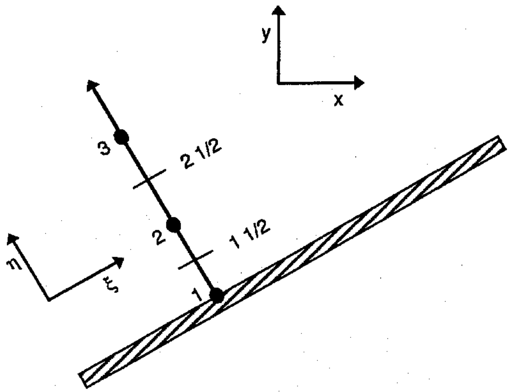

Fig. 2 Generalized curvilinear coordinate system.

solve for the Cartesian velocity components, it is the Cartesian stress tensor components, τ^{xx}, τ^{xy}, ..., that are required in the governing equations. Therefore a transformation of the stress tensor from the ξ–η coordinates to Cartesian coordinates is needed. Sondak and Pletcher proposed a very elaborate procedure for performing this formidable task. In contrast, if one defines the effective viscosity at point $1\frac{1}{2}$ according to

$$\mu_{\text{eff}} \equiv \mu_w\left(\eta_2^+ - \eta_1^+\right)/\hat{u}_2^+ \tag{8}$$

such that

$$\tau_w = \mu_{\text{eff}} \frac{\hat{u}_2 - \hat{u}_1}{\eta_2 - \eta_1} \tag{9}$$

the Cartesian stress tensor can then be obtained in a straightforward manner from

$$\tau_{1\frac{1}{2}}^{ij} = \mu_{\text{eff}}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_i}{\partial x_j} - \frac{2}{3}\delta_{ij}\frac{\partial u_k}{\partial x_k}\right) \tag{10}$$

References

[1]Sondak, D. L., and Pletcher, R. H., "Application of Wall Functions to Generalized Nonorthogonal Curvilinear Coordinate Systems," *AIAA Journal*, Vol. 33, No. 1, 1995, pp. 33–41.

[2]Huang, P. G., and Coakley, T. J., "Calculations of Supersonic and Hypersonic Flows Using Compressible Wall Functions," *Engineering Turbulence Modelling and Experiments*, edited by W. Rodi and F. Martelli, Elsevier, Amsterdam, The Netherlands, 1993, pp. 731–739.

Reply by the Authors to P. G. Huang

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THE authors would like to thank P. G. Huang for his comments on our article. He had stated that the method of Huang and Coakley (HC) is identical to that described in the article but is easier to implement. For cases in which the grid is orthogonal to the wall, the methods are indeed the same. When modeling complex geometries, however, it is often difficult to maintain grid orthogonality at solid surfaces. The method of Sondak and Pletcher (SP) was developed to handle skewed grids, eliminating one source of error.

In their current forms, neither method accounts for the buffer region. The SP method could be extended to include the buffer region, given a suitable means of deducing the shear stress in that region. Once the shear stress is known, no special treatment would be required. The HC method would require special logic for the buffer region, since both turbulent and molecular viscosities are important.

The authors believe that the simplest solution is the most appropriate solution for a given problem, and for many cases, this would lead toward the HC method. Further testing is required to evaluate differences between the two methods for cases with complex geometries, particularly when a nonorthogonal grid is employed at solid boundaries.

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